GFD 1, Problem Set #4

Due at the start of class Monday 2/27/2012

1) Consider a two layer fluid on an *f*-plane, with densities ρ_1 and ρ_2. Much like the problem in the midterm, it has a sinusoidal velocity distribution in the surface layer, given by:

$$u_1 = 0 \quad \text{and} \quad v_1 = V\cos(kx)$$

where V is a constant with units of velocity. Use the linear, Boussinesq equations for momentum and mass conservation, and assume $\rho_0 = 1000 \text{ kg m}^{-3}$.

(a) Assuming the flow is in geostrophic balance, what is the surface height field, $\eta(x)$? Note that this is not an "adjustment" problem. The flow and the density field are already in steady state.

(b) Assuming the flow in the lower layer is stationary, what is the shape of the interface displacement, $E(x)$? How big are interface displacements if $V = 0.5 \text{ m s}^{-1}$, $\rho_2 - \rho_1 = 1 \text{ kg m}^{-3}$, and $k = 2\pi/(20 \text{ km})$?

(c) Derive expressions for the depth-integrated KE_A and APE_A, assuming that the resting layer thicknesses are given by H_1 and H_2. I am looking for the fundamental expressions, in terms of u, v, η, and E. For the KE_A expression the linear form is fine, in which the depth integral is just like multiplying by the resting layer thickness, not the actual layer thickness. For the PE_A, start from the fundamental expression:

$$\frac{PE}{\text{unit area}} \equiv PE_A = \int_{-H}^{\eta} \rho g z \, dz$$

and then find the expression for the Available Potential Energy:

$$APE_A = PE_A - PE_{A0}$$

(d) What are the *x*-averaged KE_A and APE_A for the flow situation described above? Keep these in terms of V, k, g', etc., don't plug in values yet. What is their ratio $\overline{APE_A}^x \Big/ \overline{KE_A}^x$ (x-averaging is denoted by an overbar-x)? [Hint: in the x-average of APE_A, the values of the resting layer thicknesses should not appear.]

(e) The "Internal Rossby Radius of Deformation" is defined for two-layer flow as:

$$a' \equiv \frac{\sqrt{g' H_{eff}}}{f} \;, \quad \text{where} \quad H_{eff} = \frac{H_1 H_2}{H_1 + H_2} \,.$$

Simplify this to find the version in the limit of a very thick lower layer, $H_2 \to \infty$, in which case $H_{eff} \to H_1$. Now simplify your result for (d) assuming that $g >> g'$, and express your result in terms of the simplified version of a'. What is a' in this case when $H_1 = 100$ m (use $f = 10^{-4}$ s^{-1})?

(f) Defining the horizontal length scale of the flow as $L = k^{-1}$, is the flow energy mostly kinetic of potential when $L >> a'$? Is the potential energy mostly associated with surface displacements or interface displacements?